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CONCENTRATION DISTRIBUTION OF THE EXHAUST GAS NEAR A ROADWAY WITH A STOCHASTIC TRAFFIC FLOW

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The mechanical process of transfer and dispersion of the exhaust gas near a motor road has been studied based on the hypothesis of a stochastic traffic flow on the considered roadway section, which is described by a Poisson process with a constant intensity. Using the three-dimensional diffusion equation, solutions have been obtained for problems on the concentration distribution of pollutants near an extended roadway and a road crossing. Basic stochastic characteristics of the concentration distribution of the exhaust gas from a stochastic traffic flow have been determined. Calculated results have been compared with experimental data.

Introduction. The solution of the problem of evaluating the pollution of the atmosphere and soil surface near roadways is complicated by the fact that the appearance of the next vehicle on the road is of stochastic character. Additional difficulties are caused by the nonstationarity of transfer and dispersion, by the moving atmospheric air, and by impurities released by each individual moving source.

A rise in the intensity of traffic currents provided the stimulus for numerous studies devoted to the analysis of the level of pollution, by the exhaust gas, of the atmospheric air of cities. The contribution of the vehicle run exhaust to the general level of the atmospheric pollution of a modern city is assessed in [1]. In [2], various types of vehicles are considered, for which average contents of gas impurities and solid particles in the exhaust are evaluated. In [3], it is noted that the pollution by substances contained in the exhaust gas is maximum in a 100-m strip directly adjoining the road. In [4], a model of the effect of vehicles on forest ecosystems is developed, which makes it possible to investigate the pollutant concentrations and the degree of damage and dimensions of the affected zone as functions of the traffic intensity, composition of the traffic current, road parameters, and climatic and meteorological factors.

The concentration of the vehicle exhaust is as a rule determined using Gaussian models [5–8] of the pollution propagation from linear extended sources simulating sources of the exhaust gas from vehicles moving along the roadway. The solution proposed in [5] gives an overstated concentration near the roadway as compared with the data of full-scale measurements. The model [7] is used for finite-length sources, but is only applicable if the wind direction is perpendicular to the road. The model of a finite linear source, which allows for the wind direction, is considered in [8].

In [9], transfer of pollutants near the roadways is modeled using a wind tunnel, and measurement results are compared with data calculated by standard methods. Velocities of air flows and concentrations of pollutants from cars moving along city roadways are calculated in [10] with the aid of two- and three-dimensional mathematical models. The finite-element method is used in [11] for modeling advective-diffusive transfer, in the atmosphere, of pollutants from a linear infinite source, which simulated the arrival of the exhaust gas from passing vehicles.

In the current study consideration is given to one of possible approaches to evaluating the concentration field of the exhaust gas near the roadway, which is realized in the problem of transfer of the vehicle exhaust by atmospheric flows and is based on the solution of the three-dimensional diffusion equation and on the use of the hypothesis of the stochastic character of appearance of vehicles on the roadway.

Extended Section of the Roadway. We now consider an extended (in the direction of the axis Oy) section of a one-way one-lane road with the length L in the spatial region G (Fig. 1). The roadway is blown by a horizontal

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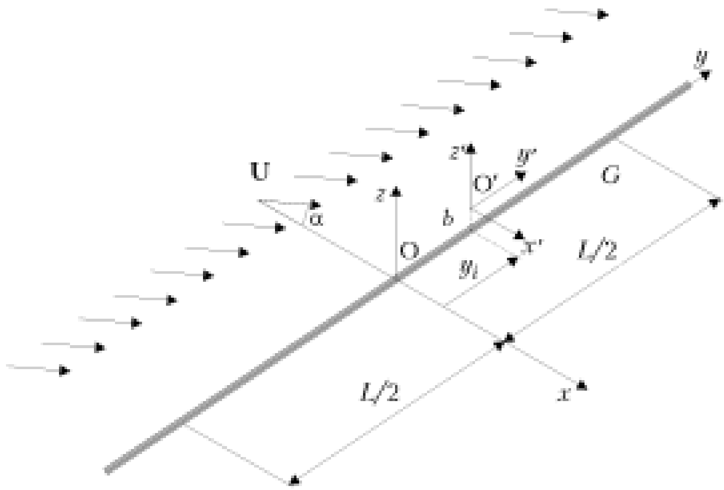

Fig. 1. Schematic of the problem of traffic flow.

air flow with a constant velocity U, which is directed to the axis Ox at the angle α. Let the velocity of the air flow at all points of the region G be independent of the disposition, speeds, and characteristics of vehicles. The concentration of pollutants near the roadway depends on the volume of impurities ejected by all vehicles, which are simultaneously situated on the considered section and are moving point sources of pollution with a constant strength q. Speeds of movement of all vehicles along the roadway are taken to be identical, constant, and equal to V. We employ the assumption that the appearance of a vehicle at the beginning of the considered roadway section is stochastic and described by the Poisson flow of events with a constant rate λ, which determines the average number of vehicles arriving at the roadway in unit time.

Determination of the concentration of an impurity from a moving point source. To date, an accurate solution has not been constructed for the problem of the concentration distribution of an impurity $\varphi(t, x, y, z)$ in the region G for arbitrary velocity fields of the air flow and a spatial distribution of diffusion coefficients K_x, K_y, and K_z. There are some approximate models, in which assumptions are used that the velocity and diffusion properties are independent of coordinates [12–14] or linear [15] and power-law [16] dependences of the wind velocity and diffusion coefficients on the vertical coordinate are adopted. The most general cases can be considered using numerical methods, since here arbitrary distributions of the input data are allowed [17–19].

The indicated solutions are mainly obtained assuming that the diffusion process is steady. Further on we consider the problem of unsteady propagation of impurities from moving sources. The use of the moving coordinate system O$'x'y'z'$ with its origin fixed at a vehicle, which moves with a constant speed (Fig. 1), makes it possible to convert to the problem described by the steady equation of diffusion of an impurity from a point source located at the point $x' = 0$, $y' = 0$, $z' = 0$ [12]

$$U_x' \frac{\partial \varphi}{\partial x} + U_y' \frac{\partial \varphi}{\partial y} + U_z' \frac{\partial \varphi}{\partial z} = \frac{\partial}{\partial x}\left(K_x \frac{\partial \varphi}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial \varphi}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial \varphi}{\partial z}\right) + q\delta\,(x', y', z') \,. \tag{1}$$

Here, the components of the relative velocity of the air flow in a moving coordinate system are defined as

$$U_x' = U\cos\alpha\,, \quad U_y' = U\sin\alpha - V\,, \quad U_z' = W\,. \tag{2}$$

Under the assumption that the wind velocity is independent of the height and the diffusion coefficients are constant, the solution of Eq. (1), as in study [12], is constructed in the form

$$\varphi\,(x', y', z') = \psi\,(x', y', z') \exp\left(\frac{U_x' x'}{2K_x} + \frac{U_y' y'}{2K_y} + \frac{U_z' z'}{2K_z}\right). \tag{3}$$

Substitution of Eq. (3) into expression (1) gives the equation

$$K_x \frac{\partial^2 \psi}{\partial x^2} + K_y \frac{\partial^2 \psi}{\partial y^2} + K_z \frac{\partial^2 \psi}{\partial z^2} - \left(\frac{U_x^{\prime 2}}{4K_x} + \frac{U_y^{\prime 2}}{4K_y} + \frac{U_z^{\prime 2}}{4K_z} \right) \psi + q\delta\,(x', y', z') = 0\,, \tag{4}$$

whose solution for the region G (under the constraint $\psi < \infty$, x', y', $z' \to \infty$) is of the form

$$\psi\,(x', y', z') = q \frac{\exp\left(-0.5 \sqrt{\frac{x^{\prime 2}}{K_x} + \frac{y^{\prime 2}}{K_y} + \frac{z^{\prime 2}}{K_z}} \sqrt{\frac{U_x^{\prime 2}}{K_x} + \frac{U_y^{\prime 2}}{K_y} + \frac{U_z^{\prime 2}}{K_z}} \right)}{4\pi \sqrt{K_x K_y K_z \left(\frac{x^{\prime 2}}{K_x} + \frac{y^{\prime 2}}{K_y} + \frac{z^{\prime 2}}{K_z} \right)}}\,. \tag{5}$$

Taking account of expressions (2) and (3) and the dependences between coordinates $x' = x$, $y' = y - y_i$, and $z' = z - b$, for the immobile coordinate system $Oxyz$ we find

$$\varphi^+\,(y_i, x, y, z) = \frac{q \exp\left[\frac{Ux \cos \alpha}{2K_x} + \frac{(U \sin \alpha - V)\,(y - y_i)}{2K_y} + \frac{W\,(z - b)}{2K_z} \right]}{4\pi \sqrt{K_x K_y K_z} \sqrt{\frac{x^2}{K_x} + \frac{(y - y_i)^2}{K_y} + \frac{(z - b)^2}{K_z}}} \times$$

$$\times \exp\left[-\frac{1}{2} \sqrt{\frac{x^2}{K_x} + \frac{(y - y_i)^2}{K_y} + \frac{(z - b)^2}{K_z}} \sqrt{\frac{U^2 \cos^2 \alpha}{K_x} + \frac{(U \sin \alpha - V)^2}{K_y} + \frac{W^2}{K_z}} \right]. \tag{6}$$

Solution (6) is obtained for an infinite region. In the considered case, the region G is bounded by a horizontal half-plane, on which the condition of absence of the impurity flow through an impenetrable surface should be fulfilled $\partial\varphi(y_i, x, y, 0)/\partial z = 0$. To allow for this boundary condition, we introduce a fictitious source positioned symmetrically to the original one with respect to the horizontal plane $z = 0$, the concentration of impurities from which is defined by the expression

$$\varphi^-\,(y_i, x, y, z) = \frac{q \exp\left[\frac{Ux \cos \alpha}{2K_x} + \frac{(U \sin \alpha - V)\,(y - y_i)}{2K_y} - \frac{W\,(z + b)}{2K_z} \right]}{4\pi \sqrt{K_x K_y K_z} \sqrt{\frac{x^2}{K_x} + \frac{(y - y_i)^2}{K_y} + \frac{(z + b)^2}{K_z}}} \times$$

$$\times \exp\left[-\frac{1}{2} \sqrt{\frac{x^2}{K_x} + \frac{(y - y_i)^2}{K_y} + \frac{(z + b)^2}{K_z}} \sqrt{\frac{U^2 \cos^2 \alpha}{K_x} + \frac{(U \sin \alpha - V)^2}{K_y} + \frac{W^2}{K_z}} \right].$$

Then, the solution of the initial problem is written in the form

$$\varphi\,(y_i, x, y, z) = \varphi^+\,(y_i, x, y, z) + \varphi^-\,(y_i, x, y, z)\,. \tag{7}$$

Stochastic characteristics of the concentration distribution of the exhaust gas. The overall concentration Φ of impurities at an arbitrary point with coordinates (x, y, z) from a random number $\tilde{N}$ of vehicles on the considered roadway section with the length L can be determined using Eq. (7):

$$\Phi(x, y, z) = \sum_{i=1}^{\tilde{N}} \varphi(y_i, x, y, z) . \tag{8}$$

The obtained analytical dependences make it possible to find the expectation $M_\Phi = \langle \Phi \rangle$ and the rms deviation $\sigma_\Phi = \sqrt{\langle (\Phi - M_\Phi)^2 \rangle}$ for the concentration Φ of vehicle impurities released at any point of the region G.

In accordance with [20], a steady Poisson flow of events in the interval $(0, t)$ can be viewed as a set of independent random points on this interval, with the coordinates $T_i \cup (0, t)$ distributed uniformly and independently. According to [21], the distances Δt between T_i are distributed in this case by the exponential law $p(\Delta t) = \lambda \exp(-\lambda \Delta t)$. Using the speed V as a scaling factor it is possible to convert to the variable $\Delta y = V\Delta t$. Then, $p(V\Delta t) = p(\Delta y) = \lambda \exp(-\lambda \Delta y/V)/V$, where Δy is the distance between vehicles on the roadway, whose expectation is

$$\langle \Delta y \rangle = \int_0^\infty \Delta y p(\Delta y) \, d(\Delta y) = \frac{\lambda}{V} \int_0^\infty \Delta y \exp(-\lambda \Delta y / V) \, d(\Delta y) = V/\lambda .$$

The length $\tilde{L}$ of a vehicle column is defined by the sum of a random number $\tilde{N}$ of random variables Δy_i, i.e., $\tilde{L} = \sum_{i=1}^{\tilde{N}} \Delta y_i$. The expectation M and the variance D of the sum of a random number $\tilde{N}$ of identically distributed random uncorrelated variables Ψ_i are defined, according to [22], by the expressions

$$M\left[\sum_{i=1}^{\tilde{N}} \Psi_i\right] = \left\langle \sum_{i=1}^{\tilde{N}} \Psi_i \right\rangle = \langle \tilde{N} \rangle \langle \Psi \rangle , \quad D\left[\sum_{i=1}^{\tilde{N}} \Psi_i\right] = \langle \tilde{N} \rangle D[\Psi] + D[\tilde{N}] \langle \Psi \rangle^2 . \tag{9}$$

Therefore, the expectation of the column length is $\langle \tilde{L} \rangle = \left\langle \sum_{i=1}^{\tilde{N}} \Delta y_i \right\rangle = \langle \tilde{N} \rangle \langle \Delta y \rangle$. At $\langle \tilde{L} \rangle = L$ the statistical mean value of the number of vehicles on the roadway is $N = \langle \tilde{N} \rangle \langle \tilde{L} \rangle / \langle \Delta y \rangle = \lambda L/V$. Since $D[\tilde{N}] = \langle \tilde{N} \rangle = \lambda L/V$, and the expectation of the exhaust concentration at the check point with allowance for expressions (8) and (9) is defined by the expression

$$M_\Phi(x, y, z) = \left\langle \sum_{i=1}^{\tilde{N}} \varphi(y_i, x, y, z) \right\rangle = \langle \tilde{N} \rangle \langle \varphi(y_i, x, y, z) \rangle = \frac{\lambda}{V} \int_{-0.5L}^{0.5L} \varphi(y_i, x, y, z) \, dy_i . \tag{10}$$

The rms deviation is found as

$$D\left[\sum_{i=1}^{\tilde{N}} \varphi(y_i, x, y, z)\right] = \langle \tilde{N} \rangle D[\varphi(y_i, x, y, z)] + D[\tilde{N}] \langle \varphi(y_i, x, y, z) \rangle^2 = \frac{\lambda L}{V} \langle \varphi^2(y_i, x, y, z) \rangle ,$$

$$\sigma_\Phi(x, y, z) = \sqrt{\frac{\lambda}{V} \int_{-0.5L}^{0.5L} \varphi^2(y_i, x, y, z) \, dy_i} . \tag{11}$$

The expectation and the rms deviation of the impurity concentration for a two-way multilane roadway (or for a roadway with various vehicles) are calculated from the equations

$$M_\Phi(x, y, z) = \sum_j \frac{\lambda_j}{V_j} \int_{-0.5L}^{0.5L} \varphi_j(y_i, x, y, z)\, dy_i\,, \quad \sigma_\Phi(x, y, z) = \sqrt{\sum_j \frac{\lambda_j}{V_j} \int_{-0.5L}^{0.5L} \varphi_j^2(y_i, x, y, z)\, dy_i}\,,$$

where $\varphi_j(y_i, x, y, z)$ is the concentration distribution of the impurity from moving sources of the jth lane (or of the jth type) having ejection power q_j, movement speed V_j, and intensity λ_j of appearance on the roadway. For the known probability density of the wind direction $p(\alpha)$, the expectation of the impurity concentration is written in the form

$$M_\Phi^\alpha(x, y, z) = \frac{\lambda}{V} \int_{-0.5L}^{0.5L} \int_0^{2\pi} \varphi(y_i, x, y, z)\, p(\alpha)\, d\alpha dy_i\,. \tag{12}$$

Expressions (10)–(12) are time-independent, which indicates the stationarity of the stochastic process of arrival of impurities released by vehicles at any point of the considered region G.

We now examine the distribution of expectation (10) of the impurity concentration in a vertical plane G_1 (at $y = 0$) equidistant from the ends of the considered roadway section. It is of interest to calculate the dimension L of the "representative" road section sufficient for finding the concentration distribution of impurities in the considered plane G_1 with adequate accuracy. For this, a functional $I(L)$ is constructed, which for two values L and $L + \Delta L$ determines the difference in two solutions (expectations) in the region G_1:

$$I(L) = \frac{\lambda}{V} \max_{x,z \in G_1} \left| \int_{-0.5L-\Delta L}^{0.5L+\Delta L} \varphi(y_i, x, 0, z)\, dy_i - \int_{-0.5L}^{0.5L} \varphi(y_i, x, 0, z)\, dy_i \right|\,. \tag{13}$$

The value of L, at which functional (13) takes nearly zero values, will allow the evaluation of the necessary minimum length of the roadway section, which can be regarded as "representative" and used for modeling the entire road.

Since the expectation of the concentration of the arriving impurity for an extended roadway section is a time-independent quantity, it seems reasonable to assess the feasibility of approximating unsteady spatial transfer and dispersion of the gas exhaust from randomly appearing vehicles using the impurity propagation from a linear stationary source with a constant strength, as is the case in studies [5–8]. For this, we employ the two-dimensional differential equation of diffusion of impurities from a pollution source continuously distributed along the road

$$U \cos\alpha \frac{\partial \phi}{\partial x} + W \frac{\partial \phi}{\partial z} = K_x \frac{\partial^2 \phi}{\partial x^2} + K_z \frac{\partial^2 \phi}{\partial z^2} + Q\delta(x, z - b) \tag{14}$$

with constraints $\phi < \infty$, $x \to \pm\infty$, $z \to \infty$, and $\partial\varphi(x, 0)/dz = 0$, where Q is the sought power of a linear source of impurity, which approximates the overall vehicle exhaust. The solution of Eq. (14) with such conditions is given in [12]:

$$\phi(x, z) = \frac{Q}{2\pi\sqrt{K_x K_z}} \left\{ K_0\left[0.5\sqrt{\frac{U^2\cos^2\alpha}{K_x} + \frac{W^2}{K_z}}\sqrt{\frac{x^2}{K_x} + \frac{(z-b)^2}{K_z}}\right] \exp\left[\frac{xU\cos\alpha}{2K_x} + \frac{(z-b)W}{2K_z}\right] + \right.$$

$$\left. + K_0\left[0.5\sqrt{\frac{U^2\cos^2\alpha}{K_x} + \frac{W^2}{K_z}}\sqrt{\frac{x^2}{K_x} + \frac{(z+b)^2}{K_z}}\right] \exp\left[\frac{xU\cos\alpha}{2K_x} - \frac{(z+b)W}{2K_z}\right] \right\}. \tag{15}$$

Expressions for the functions M_Φ and ϕ are conveniently represented as

$$M_\Phi(x, 0, z) = \frac{q\lambda\omega(x, z)}{4\pi V\sqrt{K_x K_y K_z}}\,, \quad \phi(x, z) = \frac{Q\vartheta(x, z)}{2\pi\sqrt{K_x K_z}}\,,$$

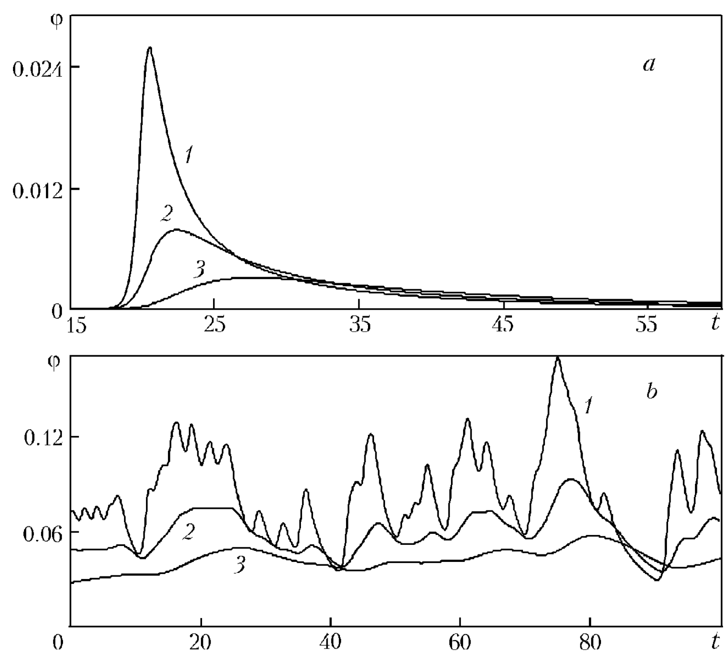

Fig. 2. Concentration of a gas impurity from a single vehicle (a) and from a stochastic traffic flow (b) at various distances from the road: 1) x = 10 m, 2) 25, 3) 50. φ, mg/m^3; t, sec.

where $\omega(x, z)$ and $\vartheta(x, z)$ are determined by comparing them with the expressions (10) and (15), respectively. The closeness of ϕ and M_Φ is evaluated using the square of the norm in G_1:

$$J = \|\phi - M_\Phi\|^2_{G_1} = (\phi - M_\Phi, \phi - M_\Phi) = \|\phi\|^2_{G_1} - 2\,(\phi, M_\Phi) + \|M_\Phi\|^2_{G_1} =$$

$$= \frac{Q^2}{4\pi^2 K_x K_z}\,\|\vartheta\|^2_{G_1} - \frac{qQ\lambda}{4\pi^2 V K_x K_z \sqrt{K_y}}\,(\vartheta, \omega) + \frac{q^2\lambda^2}{16\pi^2 V^2 K_x K_y K_z}\,\|\omega\|^2_{G_1}\,.$$

The minimum of the latter expression is reached at the value of Q at which the distribution of an impurity from a stochastic traffic flow in the considered region is best approximated:

$$\frac{dJ}{dQ} = \frac{Q}{2\pi^2 K_x K_z}\,\|\vartheta\|^2_{G_1} - \frac{q\lambda}{4\pi^2 V K_x K_z \sqrt{K_y}}\,(\vartheta, \omega) = 0\,, \quad Q = q\,\frac{\lambda}{2V\sqrt{K_y}}\,\frac{(\vartheta, \omega)}{\|\vartheta\|^2_{G_1}}\,, \tag{16}$$

where

$$(\vartheta, \omega) = \int\limits_{G_1} \vartheta\,(x, z)\,\omega\,(x, z)\,dxdz\,, \quad \|\vartheta\|^2_{G_1} = \int\limits_{G_1} \vartheta^2\,(x, z)\,dxdz\,.$$

Results of the computational experiment. In accordance with [1], the traffic intensity on city roads ranges from 450 to 1742 vehicles per hour, which corresponds to λ = 0.125–0.484 sec^{-1}. According to data of [23], the carbon oxide exhaust by cars at a speed of 45–60 km/h amounts to 9.6 g/km. The corresponding strength of a point source is q_{CO} = 0.12–0.16 g/sec. Calculations were performed assuming that the length of the considered road section is L = 1000 m; the height of the sources above the road is b = 0.5 m; the traffic intensity is λ = 0.5 sec^{-1}; the speed of vehicles is V = 12.5 m/sec; the air flow velocity is U = 3 m/sec; $\alpha = 0^{\circ}$; the rate of deposition of carbon oxide is W = 0 m/sec; at a wind velocity of 3 m/sec the turbulent diffusion coefficients are, according to [14], $K_x = K_y$ = 67 m^2/sec and K_z = 26 m^2/sec; and the strength of point sources is q_{CO} = 0.12 g/sec.

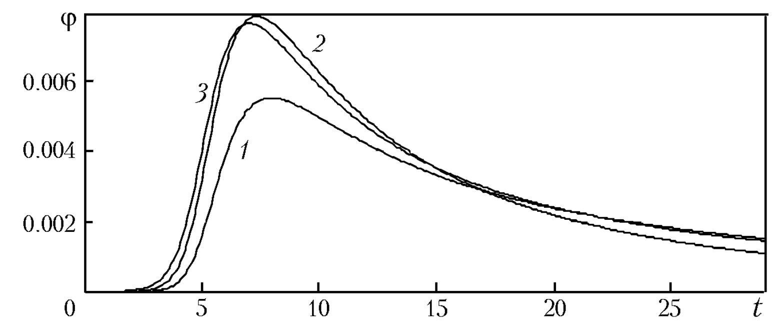

Fig. 3. Concentration of a gas impurity from a single vehicle at various wind directions: 1) $\alpha = -60^{o}$, 2) 0, 3) 60. φ, mg/m^3; t, sec.

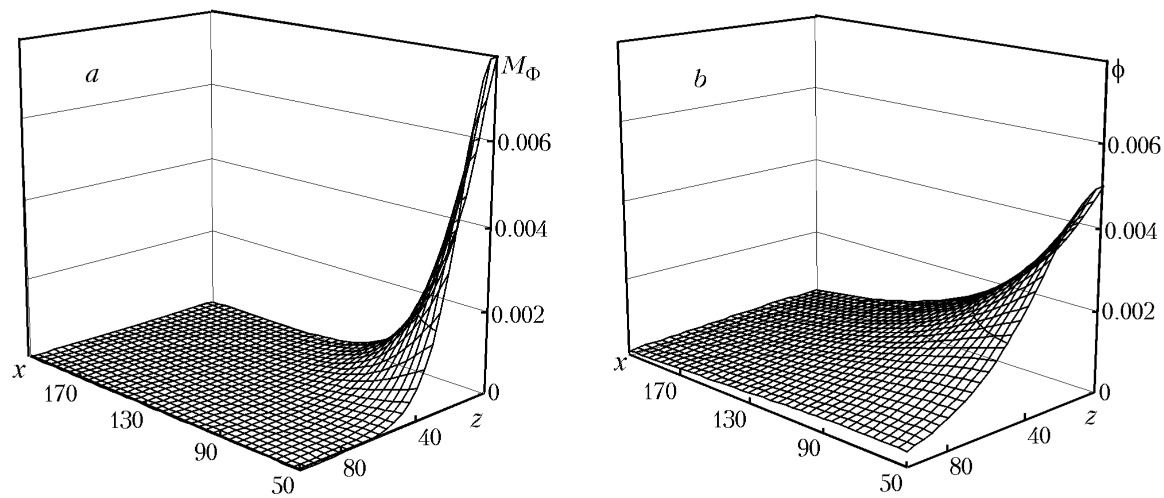

Fig. 4. Distributions of the expectation M_Φ (a) at L = 1000 m and of the concentration of an impurity ϕ from a linear stationary source (b) with the strength $Q = 1.4646 \cdot 10^{-3}$ g/(m·sec). M_Φ, mg/m^3; ϕ, mg/m^3; x, z, m.

Figures 2 and 3 present time dependences of the impurity concentration (as exemplified by a single vehicle) at check points located in the vertical plane G_1 at a height of 2 m from the surface and at the distances x = 10, 25, 50 m from the roadway (Fig. 2a) and also at various wind directions (for the distance x = 25 m, Fig. 3). For the same points, time dependences of the impurity concentrations for a stochastic traffic flow are determined. The arrival of vehicles at the initial point of the roadway is simulated by a steady Poisson process. For each specific instant of time, the position, on the road, of each point source of impurity with the strength q_{CO} is established from the known speed V and the corresponding concentration field of the gas exhaust is determined, after which these fields are summed up (a linear problem is considered). The impurity concentrations at arbitrary check points (and therefore, at all the other points of the region) are described by random functions, whose individual realizations are presented in Fig. 2b.

Calculations of the representative dimension L of the roadway section show that the values of the functional $I(L)$ = 0.000889 mg/m^3 and $I(L + \Delta L)$ = 0.000881 mg/m^3 already at the length L = 1000 m and the increment ΔL = 10 m differ by no more than 1%. Therefore, with accuracy acceptable for modeling a steady arrival of an impurity at the considered region it is sufficient to examine a section with the length L = 1000 m. The distribution of the expectation M_Φ of the impurity concentration on the part of the region G_1 at such L is given in Fig. 4a.

Using expression (16) it is possible to assess the feasibility of approximating the expectation M_Φ, which is defined by expression (10), by the concentration distribution of an impurity from a linear stationary source with the constant strength Q. Calculations for the region G_1 give $Q = 1.4646 \cdot 10^{-3}$ g/(m·sec) (Fig. 4b). Comparison of the results (Fig. 4) indicates that we failed to find an identical replacement of the spatial distribution of M_Φ of impurities from a stochastic traffic flow by the model with a linear source with the constant strength Q.

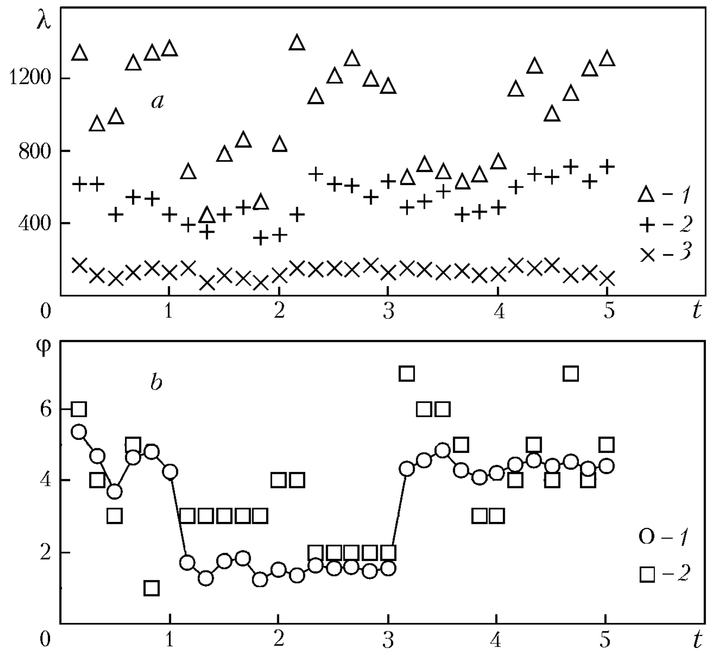

Fig. 5. Intensity λ of traffic flow (a) (1, cars; 2, trucks; and 3, buses) and calculated and experimental values of the concentration φ of carbon dioxide in the exhaust gas of vehicles (b) (1, calculated values; 2, experimental values) as functions of the time t. λ, unity/h; φ, mg/m^3; t, day.

TABLE 1. Velocities and Directions of Wind near a Roadway in Measurements of Carbon Oxide Concentrations (October 1998)

Day of observation	Wind direction, deg	Wind velocity, m/sec
1	80	6
2	70	3
3	100	5
4	80	2
5	50	1

The data of full-scale measurements of the carbon oxide concentration near a main road with heavy traffic[*)] are compared with the calculated results (Fig. 5). Figure 5a shows experimental time dependences for the intensity of movement of vehicles with a different power of ejection of the exhaust gas, and Fig. 5b presents calculated results and experimental data for the carbon oxide concentration. Data on meteorological characteristics of the atmosphere during measurements are supplied in Table 1.

For the point with coordinates $x = 10$ m, $y = 0$ m, and $z = 2$ m, with the aid of the developed model we determined the mean concentration $\overline{\Phi} = \frac{1}{t}\int_0^t \Phi(\tau)d\tau$ and the rms deviation $s_\Phi = \sqrt{\frac{1}{t}\int_0^t (\Phi(\tau) - \overline{\Phi})^2 d\tau}$ of the impurity concentration from the mean value $\overline{\Phi}$ using realizations of the stochastic process of a one-way single-lane movement (Fig. 6). These results demonstrate the convergence $\overline{\Phi} \to M_\Phi$, $s_\Phi \to \sigma_\Phi$ with increasing t and allow the evaluation of the time at which the concentration reaches a mean value close to the true value of the expectation. For the considered conditions it is about 35 min (the $\overline{\Phi}$ deviation from M_Φ is no more than 2.5%), i.e., in exactly this interval of time

[*)]Data of the State Committee for Environmental Protection of the Perm region.

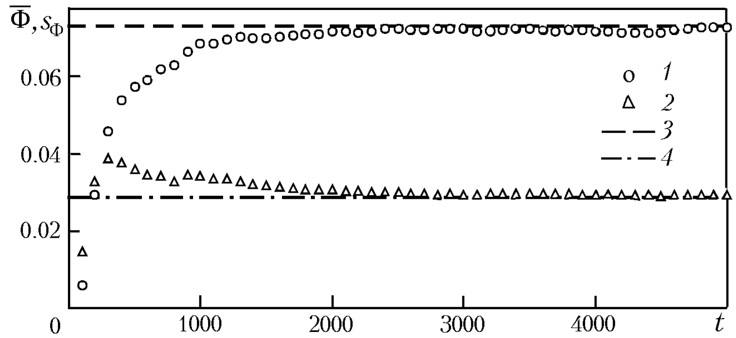

Fig. 6. Convergence of the values of the mean concentration $\overline{\Phi}$ (1) and the rms deviation s_Φ (2) to the accurate values of M_Φ (3) and σ_Φ (4) with time. $\overline{\Phi}$, mg/m^3; s_Φ, mg/m^3; t, sec.

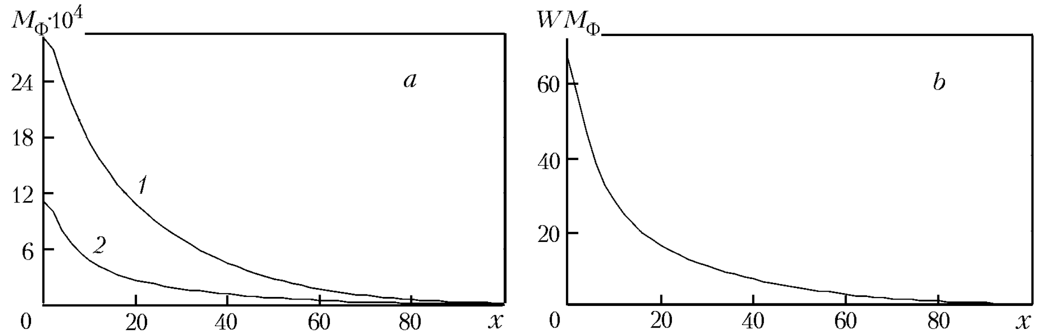

Fig. 7. Expectation M_Φ (a) of the concentration of a heavy impurity at a steady wind (1) and with account for the wind rise (2) and the annual deposition WM_Φ of a heavy impurity on the adjacent territory (b) as functions of the distance x to the roadway. M_Φ, mg/m^3; WM_Φ, kg/km^2; x, m.

from the beginning of movement of the first vehicle it is necessary to observe the impurity concentration in order to obtain a mean concentration close to the value of the expectation.

On the example of lead compounds we examine transfer, dispersion, and sedimentation of heavy impurities emitted by vehicles from a four-lane roadway. According to [23], we set q_{Pb} = 0.0012 g/sec and λ = 2.0 sec^{-1}, and the other parameters remain the same. The mean rate of sedimentation of heavy particles containing lead is estimated by G. M. Chernogaeva et al. [24] as W = –0.002 m/sec. Relative frequencies of the wind direction over a year (the wind rise relative to the considered road section, [25]) at α = 45, 90, 135, 180, 225, 270, 315, and 360° were 0.02, 0.10, 0.06, 0.12, 0.05, 0.24, 0.10, and 0.16, respectively. On the remaining days the weather was windless.

Distributions of the expectations of the lead concentration over the soil surface (z = 0) at a constant wind direction (α = 0) and with account for the indicated wind rise are given in Fig. 7a. The flux of heavy particles to the soil is defined by the product $WM_\varphi^\alpha(x, 0, 0)$. This allows the evaluation of an annual deposition of lead compounds on the soil area adjoining the road (Fig. 7b). According to [24, 26], the lead concentrations in the soil range from 1.1–2 kg/km^2 in a rural area to 300 kg/km^2 in cities with dense traffic.

Two-Road Crossing. We now consider a crossing of two road sections in the spatial region G (Fig. 8). As previously, the roadway is blown by a horizontal air flow having a constant velocity U and directed to the axis Ox at the angle α. All other assumptions as to the character of movement of vehicles remain the same. We are to find the concentration distribution, in the region G, of impurities released by vehicles near the crossing.

Determination of the concentration of the exhaust gas. Since in the case in question the hypothesis of constant speed of vehicles is not acceptable, consideration is given to a nonstationary version of the diffusion equation [12]

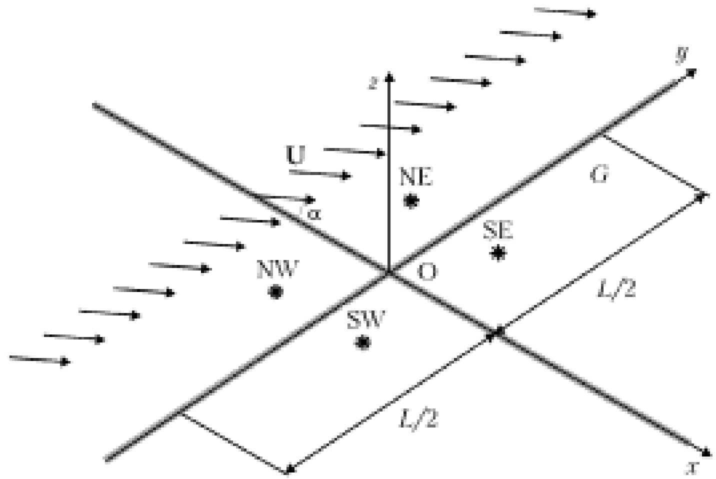

Fig. 8. Schematic of the problem of traffic flow (x, "north–south"; y, "west–east") near a road crossing (NE, SE, SW, and NW are check points).

$$\frac{\partial\varphi}{\partial t} + U_x\frac{\partial\varphi}{\partial x} + U_y\frac{\partial\varphi}{\partial y} + U_z\frac{\partial\varphi}{\partial z} = K_x\frac{\partial^2\varphi}{\partial x^2} + K_y\frac{\partial^2\varphi}{\partial y^2} + K_z\frac{\partial^2\varphi}{\partial z^2} + q\delta\,(t - t_s, x - x_s, y - y_s, z - b)\,. \tag{17}$$

Here, t_s is the moment of the instantaneous ejection of a cloud of the exhaust gas by a vehicle located at the point with coordinates (x_s, y_s, b). As in study [14], the solution of Eq. (17) is found in the form

$$\varphi\,(t, x, y, z) = \psi\,(t, x, y, z)\,\exp\left[\frac{U_x\,(2x - U_x t)}{4K_x} + \frac{U_y\,(2y - U_y t)}{4K_y} + \frac{U_z\,(2z - U_z t)}{4K_z}\right].$$

Substituting this relation into Eq. (17) results in the equation

$$\frac{\partial\psi}{\partial t} = K_x\frac{\partial^2\psi}{\partial x^2} + K_y\frac{\partial^2\psi}{\partial y^2} + K_z\frac{\partial^2\psi}{\partial z^2} +$$

$$+\,q\delta\,(t - t_s, x - x_s, y - y_s, z - b)\,\exp\left[-\frac{U_x\,(2x_s - U_x t_s)}{4K_x} - \frac{U_y\,(2y_s - U_y t_s)}{4K_y} - \frac{U_z\,(2b - U_z t_s)}{4K_z}\right],$$

whose solution is

$$\psi\,(t, x, y, z) = \frac{A}{(t - t_s)^{3/2}}\exp\left[-\frac{(x - x_s)^2}{4K_x\,(t - t_s)} - \frac{(y - y_s)^2}{4K_y\,(t - t_s)} - \frac{(z - b)^2}{4K_z\,(t - t_s)}\right],$$

where

$$A = \frac{q}{8\pi\,\sqrt{\pi K_x K_y K_z}}\exp\left[-\frac{U_x\,(2x_s - U_x t_s)}{4K_x} - \frac{U_y\,(2y_s - U_y t_s)}{4K_y} - \frac{U_z\,(2z_s - U_z t_s)}{4K_z}\right].$$

Finally, with allowance for the boundary condition $\partial\varphi(y_i, x, y, 0)/\partial z = 0$, the solution of Eq. (17) is of the form

$$\varphi\,(t, x, y, z) = \varphi^+\,(t, x, y, z) + \varphi^-\,(t, x, y, z)\,,$$

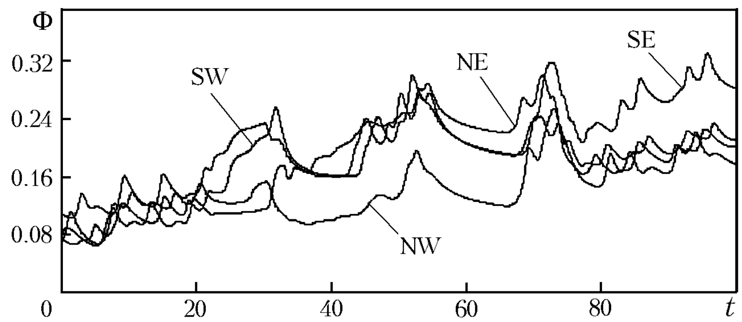

Fig. 9. Time dependences of the impurity concentration at checkpoints near a road crossing (see Fig. 8). Φ, mg/m^3; t, sec.

$$\varphi^{\pm}(t, x, y, z) = \frac{q}{8\pi (t - t_s) \sqrt{\pi (t - t_s) K_x K_y K_z}} \times$$

$$\times \exp\left\{-\frac{[(x - x_s) - U_x (t - t_s)]^2}{4K_x (t - t_s)} - \frac{[(y - y_s) - U_y (t - t_s)]^2}{4K_y (t - t_s)} - \frac{[(z \pm b) - U_z (t - t_s)]^2}{4K_z (t - t_s)}\right\}.$$

For a vehicle situated on the roadway section for the time τ, the concentration field of an impurity is defined by the expression

$$\theta(t, x, y, z) = \theta^{+}(t, x, y, z) + \theta^{-}(t, x, y, z),$$

where

$$\theta^{\pm}(t, x, y, z) = \frac{q}{8\pi \sqrt{\pi K_x K_y K_z}} \times$$

$$\times \int_0^{\tau} \frac{\exp\left\{-\frac{[(x - x_s) - U_x (t - t_s)]^2}{4K_x (t - t_s)} - \frac{[(y - y_s) - U_y (t - t_s)]^2}{4K_y (t - t_s)} - \frac{[(z \pm b) - U_z (t - t_s)]^2}{4K_z (t - t_s)}\right\}}{(t - t_s)^{3/2}} dt_s . \qquad (18)$$

It can be shown that, under appropriate assumptions, solution (6) for an extended roadway section is a specific case of expression (18).

The concentration of the exhaust gas from a stochastic current of vehicles simultaneously located near the road crossing can be defined as

$$\Theta(t, x, y, z) = \sum_{i=1}^{\tilde{N}} \theta_i (t, x, y, z).$$

Results of the computational experiment. As in the previous case, it is assumed that the sources are at the height b = 0.5 m above the road, the intensity of traffic currents in all directions is λ = 0.5 sec^{-1}, the speeds of vehicles before and after the crossing are V = 12.5 m/sec, the velocity of the air flow is U = 3 m/sec, $\alpha = 0^{\circ}$, the coefficients of turbulent diffusion are $K_x = K_y$ = 67 m^2/sec and K_z = 26 m^2/sec, and the strength of point sources (both moving and standing before the traffic light) is q_{CO} = 0.12 g/sec. The traffic lights (in each direction) operate in the following mode: for 30 sec, go, for 30 sec, stop, and for 10 sec, wait. During the computational experiment, to obtain

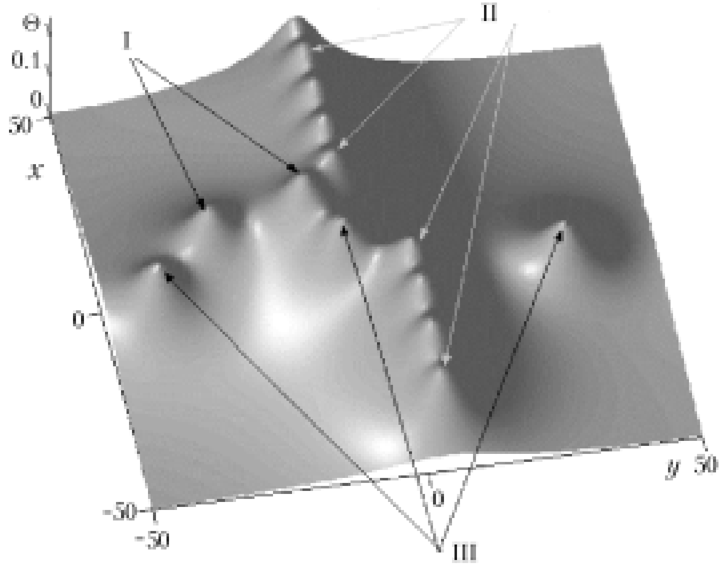

Fig. 10. Concentration distribution of the exhaust gas from a stochastic traffic flow near a road crossing (at a height of 2 m); I, "eastern" traffic current; II, vehicles waiting at a road crossing; III "western" traffic current. Θ, mg/m^3; x, y, m.

the concentrations at each point (x, y, z) of the region G the value of the integral in expression (18) was determined numerically.

The arrival of vehicles at the initial points of two roads of the crossing is modeled by a steady Poisson process. For each instant of time, the position, on the road (including the stop before the crossing at a "red" signal of the traffic light), of each point source of impurity with the strength q_{CO} is established from the known speed V and the corresponding concentration field of the exhaust gas is determined, after which these fields are summed up in order to obtain the total concentration of the exhaust gas in the atmosphere.

Figure 9 presents individual realizations of the considered process at four check points marked in Fig. 8 (at a height of 2 m above the surface) for a stochastic traffic flow near a signaled crossing. Figure 10 gives the concentration distribution of the exhaust gas near the crossing at a height of 2 m above the ground surface.

Conclusion. Solutions are presented for the three-dimensional problem of transfer and dispersion of the exhaust gas from a stochastic Poisson flow of vehicles near an extended roadway section and a road crossing. Expressions for the expectation and rms deviation of concentrations of the exhaust gas are found, which depend on the point of observation, length of the roadway section, wind direction, intensity of traffic currents, and other factors. It is shown that the expectation of the concentration of arriving impurities for extended road sections and long time intervals is a stationary quantity. Obtaining an identical approximation of the process of arrival of impurities from randomly appearing vehicles using the model with a continuously distributed constant-strength source of impurities failed. The presented model made it possible to determine the time of performing full-scale measurements to find the concentration of the exhaust gas and the length of the road section that may be viewed as "representative."

NOTATION

b, height of the source of impurity; D, variance of a random variable; I and J, functionals; $K_0(\bullet)$, Macdonald function; K_x, K_y, and K_z, diffusion coefficients in the directions of coordinate axes; L, length of a vehicle column (of a roadway section); $\tilde{L}$, random length of a vehicle column; M, expectation of a random variable; M_Φ, expectation of the overall impurity concentration Φ; $\tilde{N}$, random number of vehicles; p, probability density; q, strength of a point source of impurity; q_{CO}, strength of a point source of carbon oxide; q_{Pb}, strength of a point source of lead compounds; Q, strength of a linear source of impurity; s_Φ, rms deviation of the overall impurity concentration Φ; t, time; t_s, moment of the instantaneous gas exhaust; T_i, moments of the appearance of vehicles on the roadway; $\mathbf{U}$ and U, vector and modulus of the wind velocity; U_x', U_y', and U_z', components of the wind velocity vector; V and V_j, speed (speeds) of a vehicle; W, vertical velocity of an impurity; x_s and y_s, coordinates of an instantaneous source of the gas; α, angle of the wind direction; δ, Dirac function; Δt, interval between the appearances of vehicles; Δy, distance between vehicles; ϕ, function describing the impurity concentrations in the plane G_1; φ, φ_j, φ^+, and φ^-, functions describing the impurity concentrations; Φ, overall impurity concentration; $\bar{\Phi}$, mean impurity concentration; θ, θ^+, and θ^-, functions describing the impurity concentrations; Θ, overall concentration of the exhaust gas; ϑ, ω, and ψ, auxiliary functions; λ and λ_i, traffic intensity (intensities); σ_Φ, rms deviation of the overall impurity concentration Φ; Ψ_i, random uncorrelated quantities. Subscripts: s, source.

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